Power in the Sinusoidal Steady State

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Let's consider the power of a linear circuit in the sinusoidal steady state. We know that if a linear circuit is driven with a sinusoidal input, the voltage across and the current through the circuit be represented as

$$V = V_0 e^{j(\omega t + \delta_1)} \tag{1}$$

$$I = I_0 e^{j(\omega t + \delta_2)} \tag{2}$$

Let's reduce the equation for power

$$P = Re\{V\} Re\{I\} \tag{3}$$

with the voltage and current above.

$$P = (\frac{V + \bar{V}}{2}) \times (\frac{I + \bar{I}}{2})$$

$$P = \frac{1}{4}(VI + \bar{V}\bar{I} + V\bar{I} + I\bar{V})$$

$$P = \frac{1}{2}(Re\{IV\} + Re\{I\bar{V}\}) \tag{4}$$

This equation is the one to remember, it is useful in further derivations (then again, it is so simple to derive this equation itself...). This expression for instantaneous power has value

$$P = \frac{1}{2}(Re\{I_0 V_0 e^{j(2\omega t + \delta_1 + \delta_2)}\} + Re\{I_0 V_0 e^{j(\delta_2 - \delta_1)}\})$$

$$P = \frac{1}{2} I_0 V_0 (\cos(2\omega t + \delta_1 + \delta_2) + \cos(\delta_2 - \delta_1)) \tag{5}$$

Since the first term of the equation has an average value of 0 over one period, the average power, $\bar{P}$, is

$$\bar{P} = \frac{1}{2} I_0 V_0 \cos(\delta_2 - \delta_1) \tag{6}$$